REDUCED MATERIAL MODEL OF COMPOSITE LAMINATES FOR 3D FINITE ELEMENT ANALYSIS

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ABSTRACT

Laminate composites are widely used in automotive, aerospace, medical, and increasingly in consumer industries, due to their reduced weight, superior structural properties and cost-effectiveness. However, structural analysis of complex laminate structures remains challenging. 2D finite element methods based on plate and shell theories may be accurate and efficient, but they generally do not apply to the whole structure, and require identification and preprocessing (dimensional reduction) of the regions where the underlying assumptions hold. Differences in and limitations of theories for thin/thick plates and shells further complicate modeling and simulation of composites. Fully automated structural analysis using 3D elements with sufficiently high order basis functions is possible in principle, but is rarely practiced due to the significant increase in computational integration cost in the presence of a large number of laminate plies.

We propose to replace the actual layup of the laminate structure by a simplified material model, allowing for a substantial reduction of the computational cost of 3D FEA. The reduced model, under the usual assumptions made in lamination theory, has the same constitutive relationship as the corresponding 2D plate model of the original laminate, but requires only a small fraction of computational integration costs in 3D FEA. We describe implementation of 3D FEA using the reduced material model in a meshfree system using second order B-spline basis functions. Finally, we demonstrate its validity by showing agreement between computed and known results for standard problems.

1 Introduction
1.1 Motivation

Composite laminates are made of plies stacked and fused together to form complex light-weight and stiff engineering parts [1–3]. High stiffness is achieved by parallel fiber reinforcements embedded within each ply, which also make plies anisotropic in their material properties. Laminates are very attractive as design materials as their structural properties can be customized to a great extent by controlling fiber direction within each ply as well as the number of plies, but the presence of numerous thin-layered, anisotropic plies with discrete material changes across ply interfaces leads to complex deformation and stress-strain fields in laminate structures. Assuming that adjacent plies are permanently fused together, a common practice in structural analysis of laminates is to ignore inter-ply phenomena and focus on the global stress-strain and deformation [1, 2, 4, 5]. We note, however, that the inter-layer stress-strains can be partially predicted from global deformation [1].

Locally, laminates are usually thin in the layup direction relative to the overall size of the structure and tend to deform like
plates or shells. Different assumptions about the strain field in the thickness direction lead to a number of lamination theories, including Classical Laminated Plate Theory, First-Order Laminated Plate Theory, and others [2, 4, 5]. The most commonly assumed strain field for plate stretching and bending problems is a linear field (Figure 1 C). For thick plates with thickness comparable to other dimensions, shear stress in thickness direction becomes significant and may be captured using higher order lamination theories [4, 7–9]. 2D FEA methods based on these theories are generally quite accurate and efficient for both thin and thick plates, but their application is limited to specifically created idealizations and simplifications of real structures. Specifically, plate and shell theory assumptions require abstracting the complex 3D parts by “mid-surfaces,” often an ill-defined task that require substantial expertise, manual preprocessing, or may not be possible [10]. Creating plate assemblies and interfacing them with 3D models leads to additional modeling difficulties [5]. Even for simple laminates, 2D FEM based on higher order theories shows numerical locking when used for very thin plates [11]. As a result, multiple theories are often needed in the same FEM system, and user must have technical know-how in order to identify a suitable theory for a given problem. Perhaps most importantly, laminates, unlike usual plate-shell structures, can be complex monolithic parts with joints and ply transitions [2, 3, 12]. Plate theory assumptions do not generally apply everywhere within such parts, further limiting usefulness of 2D FEA methods.

Unlike 2D FEA methods, 3D FEA doesn’t require special theories, and is general and robust [13], at least in principle. However, using 3D FEA for thin structures such as laminates is often considered impractical. Meshing each ply as an independent solid body requires a prohibitively large number of elements. As a compromise, several hybrid methods that incorporate both 2D and 3D FEA have been proposed. For example, **Solid-shell elements** (Figure 2A) constrain degrees of freedom on the top and bottom surfaces of a solid element to deform like a plate [5]. Their 3D nature is well suited for interfacing with other solid elements in assemblies. Of course, such elements are still based on 2D lamination theories, and all other plate theory restrictions still apply. In 3D layered element method, multiple plies can cross an element (Figure 2B), significantly reducing the number of elements required for FEA of laminates [14]. The material properties of plies within each element are homogenized, but sufficient degrees of freedom and full integration over all the plies ensure accurate global deformation and stress-strains fields [15]. Since **layered elements** have no built-in assumptions about stress or strain fields, they remain valid everywhere in the composite laminate structure, whether plate theory assumptions apply or not. On the other hand, using 3D elements for analysis of thin structures may lead to ill-conditioned stiffness matrices, and homogenization over large number of plies may become computationally expensive. For completeness, we mention that several other methods have been proposed [4, 7–9], but they either lack generality, or suffer from one or more of the difficulties mentioned above.

### 1.2 Problem Identification and Contributions

Based on the above discussion, it is apparent that a general and fully automated analysis of composite structures is most likely to be achieved using 3D layered elements. This in turn requires addressing the two difficulties identified above. First, 3D integration over thin structures may result in shear locking and an ill-conditioned stiffness matrix [13, 16], but these problems can be alleviated or removed by using higher order hierarchical basis functions [17, 18].

The second and the main drawback of 3D FEA layered elements is the high cost of numerical integration required for homogenizing material properties of large number of plies (tens or even hundreds) present within each element. The specific in-

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1A basis function is called hierarchical when a higher order basis function contains all the lower order basis functions; for example, B-splines are hierarchical basis functions.
tegration quadrature rule [13] depends on the geometry of the element and degree of the integrand, and volume integration amounts to sampling the integrand at a number of quadrature points. To get an idea of the increase in cost of integration for laminates, take a simple example of a laminate with 100 plies that are homogenized within a linear brick element. To fully integrate a linear brick element with constant material coefficients, 8 integration points are needed [13]. However, in our case, 100 plies with different material properties are intersecting this element, which implies that we need 8 integration points for each ply, leading to a 100-fold increase in the computation cost. Since integration cost represents a significant portion of the overall solution procedure, analysis of composite laminates using 3D layered elements is an expensive proposition.

Our goal is to make 3D FEM a viable method for laminates by reducing this extra cost of integration. Specifically, we propose a novel reduced material model of composite laminates that significantly improves the efficiency of using general 3D layered finite elements. The reduced model is a simple 3-ply laminate that is equivalent to the original multi-ply laminate, under the usual plate theory assumptions. To demonstrate the efficacy of the reduced model, we implement it in a meshfree system using second-degree B-splines basis functions, and verify its accuracy on several benchmark problems.

1.3 Outline

The rest of the paper is organized as follows. In Section 2, we briefly summarize the relevant theories and derivations used to establish equivalence between the full and the reduced material model of composite laminates. A prototype implementation in a meshfree setting is described in Section 3. It is followed by numerical validation and comparison with known benchmarks and solutions computed by commercial FEA codes in Section 4.

2 Material Models for Composite Laminates

In this section, we briefly summarize the classical theories used to establish the constitutive relationships in a laminate, and derive an equivalent reduced material model which remains valid in a general 3D layered element under identical assumptions.

2.1 Constitutive Relations for Orthotropic Plies

In linear elasticity, stiffness matrix \( \mathbf{C} \) is used to characterize a material. Since plies are orthotropic in nature, the corresponding matrix \( \mathbf{C} \) requires 9 independent elastic constants [1]. The constitutive relation between stress and strain takes a general form given by [1]:

\[
\sigma_i = C_{ij} \cdot \varepsilon_j \quad i, j = 1, 2, \ldots, 6,
\]

where \( \sigma \) is stress and \( \varepsilon \) is strain, as usual. The equation is in contracted notation where \( i, j = 1, 2, 3 \) are \( x, y, z \) frame directions while \( i, j = 4, 5, 6 \) are \( yz, zx, xy \) planes respectively. Note that, unlike for isotropic materials, \( C_{ij} \) is direction dependent.

The plane-stress constitutive relationship for dimensionally reduced thin laminates is characterized by a 3x3 stiffness matrix \( \mathbf{Q} \) [1]. \( \mathbf{Q} \), so that

\[
\sigma_i = Q_{ij} \cdot \varepsilon_j \quad i, j = 1, 2, 3
\]

where \( i, j = 1, 2, 3 \) and they stand for \( x, y \) directions and \( xy \) plane respectively.

For thick plates, out-of-plane shear stiffness, in addition to in-plane stiffness, is needed to characterize a ply. Out-of-plane shear stresses \( \sigma_{yz} \) and \( \sigma_{xz} \) are significant, and, in arbitrary coordinate system, are related to shear strain \( \varepsilon_{yz} \) and \( \varepsilon_{xz} \) by [4]:

\[
\begin{bmatrix}
\sigma_4 \\
\sigma_5 \\
\end{bmatrix} = \begin{bmatrix}
Q_{44} & Q_{45} \\
Q_{45} & Q_{55} \\
\end{bmatrix} \cdot \begin{bmatrix}
\varepsilon_4 \\
\varepsilon_5 \\
\end{bmatrix},
\]

where indices 4 and 5 stand for planes \( yz \) and \( xz \) respectively.

Let us now assume that \( z \) direction is the thickness direction, which is the same for both laminate and individual plies and is also aligned to third principal direction of the orthotropic ply materials. Recall that plate theory assumes that the thickness of a plate in stretching and pure bending remains constant, or in other words, that Poisson’s ratios \( \nu_{yz} \) and \( \nu_{xz} \) are zeros [5]. These assumptions reduce the general stress-strain Equation (1) to

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{xz} \\
\sigma_z \\
\end{bmatrix} = \begin{bmatrix}
Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\
Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\
Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\
0 & 0 & Q_{44} & Q_{45} & 0 & 0 \\
0 & 0 & Q_{45} & Q_{55} & 0 & 0 \\
0 & 0 & 0 & 0 & E_3 & 0 \\
\end{bmatrix} \cdot \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{xz} \\
\varepsilon_z \\
\end{bmatrix}
\]

We have reordered the indices of \( C_{ij} \) to match those of \( Q_{ij} \). Also, \( C_{33} \) reduces to Young’s Modulus in third principal material direction \( E_3 \).

2.2 Lamination Theories

The classical lamination theory is based on the theory of plates, which assumes that laminates can only undergo stretching and pure bending. Therefore, strain at any point in the laminate can be related linearly to strain \( \varepsilon'_i \) and curvature \( \kappa_i \) at the mid-plane by (5) [1]:

\[
\varepsilon_i = \varepsilon'_i + z \cdot \kappa_i \quad i = 1, 2, 3.
\]
Figure 1C shows the plot of $\varepsilon_1$ at a typical cross section of a laminate.

To get a dimensionally reduced model for a laminate, the stresses are integrated over the thickness $t$ of the plate from $-\frac{t}{2}$ to $\frac{t}{2}$ into forces $N_i$ and moments $M_i$ on the midplane respectively:

$$N_i = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_i dz, \quad M_i = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_i zdz.$$  \hspace{1cm} (6)

Combining this integration with plane-stress constitutive relationship (Equation (2)) and pure bending deformation (Equation (5)), leads to the so-called ABD matrix model of midplane behavior for laminates:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_o \\ \kappa \end{bmatrix},$$ \hspace{1cm} (7)

where the individual coefficients are obtained by integrating $Q_{ij}$, $i, j = 1, 2, 3$:

$$A_{ij} = \int_{-\frac{t}{2}}^{\frac{t}{2}} Q_{ij} dz, \quad B_{ij} = \int_{-\frac{t}{2}}^{\frac{t}{2}} Q_{ij} zdz, \quad D_{ij} = \int_{-\frac{t}{2}}^{\frac{t}{2}} Q_{ij} z^2 dz.$$ \hspace{1cm} (8)

Intuitively, $A$ and $D$ are extensional and bending components of stiffness respectively, while $B$ couples stiffness between bending and stretching which occurs due to asymmetry of the material properties about the mid-plane. If $B$ is a non-zero matrix, a normal pull (in $x$ or $y$ direction) can lead to bending and vice versa.

The manner in which in-plane stresses were reduced to forces $N_i$ in Equations (6) and (7), out-of-plane shear stresses can also be reduced to shear forces $\Gamma_4$ and $\Gamma_5$:

$$\begin{bmatrix} \Gamma_4 \\ \Gamma_5 \end{bmatrix} = K \cdot \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{bmatrix},$$ \hspace{1cm} (9)

which assumes that $\varepsilon_4$ and $\varepsilon_5$ are constant in $z$ direction, and uses a correction factor $K$ to compensate for any deviation from the actual field [4]. The extensional shear stiffness coefficients $A_{44}, A_{45}, A_{55}$ are again defined as $A_{ij}$ in Equation (8).

### 2.3 The ABD-Equivalent Reduced Model of Laminate

If we accept the ABD-matrix model as an accurate approximation of the laminate’s behavior, it stands to reason that any two material models resulting in identical ABD matrices should be deemed equivalent. We now seek the simplest such model, with the smallest possible number of plies from the equivalence class of all laminates for a given ABD matrix. In other words, we require that

$$A_{ij}^o = \int_{-\frac{t}{2}}^{\frac{t}{2}} Q_{ij}^o dz = \int_{-\frac{t}{2}}^{\frac{t}{2}} Q_{ij}^j dz, \quad B_{ij}^o = \int_{-\frac{t}{2}}^{\frac{t}{2}} Q_{ij}^o zdz = \int_{-\frac{t}{2}}^{\frac{t}{2}} Q_{ij}^j zdz, \quad D_{ij}^o = \int_{-\frac{t}{2}}^{\frac{t}{2}} Q_{ij}^o z^2 dz = \int_{-\frac{t}{2}}^{\frac{t}{2}} Q_{ij}^j z^2 dz,$$ \hspace{1cm} (10)

where the original laminate with material properties $Q_{ij}^j$ defines coefficients $A_{ij}^o, B_{ij}^o$ and $D_{ij}^o$, but we seek a new laminate with material properties $Q_{ij}^o$ which, when integrated over the thickness of the laminate, will yield the same ABD coefficients.

Since material properties $Q_{ij}$ remain constant with each ply, the above integral equations reduce to a system of three linear equations: for extensional stiffness $A$, coupling stiffness $B$ and bending stiffness $D$. It should be apparent that each $Q_{ij}$ contributes to exactly three such linear equations and hence is completely determined by material properties of exactly three plies $Q_{ij}^k$, $k = 1, 2, 3$, which can be uniquely determined by solving the system of 3 linear equations in Equation (10). Figure 3 shows a 3-ply laminate with plies $Q_{ij}^k$ which is ABD-equivalent to the original $n$-ply laminate with plies $Q_{ij}^j$. A system of linear equations has to be solved for each $Q_{ij}^k$, but due to symmetry, the number of off-diagonal $Q_{ij}^j$ is reduced to half.

The coefficients $Q_{ij}^k, i, j = 1, \ldots, 5$, for all the three new plies in the reduced laminate model can be obtained using Equation 10, but we still need Young’s modulus $E_3^j$ in Equation (4) in order to fully characterize the behavior of each ply in the $z$ direction. It is well known that for thin layered structures, the resultant out-of-plane Young’s Modulus $E_3^j$ can be approximated as the harmonic average of the Young’s Modulus $E_{35}^j$ of the individual plies [1, 19]. Since we are only interested in the equivalent material behavior, $E_3^j$ can be assumed constant for all three plies.
and can be obtained from equivalence relation

\[
\frac{1}{E_{33}} = \sum_{k=1}^{h} \frac{h^k}{E_{33}^{ok}} = \sum_{k=1}^{h} \frac{h^{*k}}{E_{33}^{sk}}
\]

where \(h^k\) and \(h^{*k}\) are the thickness of original and equivalent plies respectively.

To summarize, a new 3-ply laminate model can be efficiently constructed for an arbitrary laminate by solving linear Equations 10 and 11. The solution has to be computed once for each coefficient in the material model. Once computed, this new ABD-equivalent 3-ply laminate can be used for computation whenever the assumptions of the laminate model apply. In the next two sections, we briefly discuss implementation of this new model and compare its results with results from existing methods for some benchmark problems.

3 Implementation

This new reduced laminate model can be implemented in any 3D FEA system which supports composite laminates. The current and emerging standards propose that laminates are commonly defined as a base surface and an associated layup table with \(n\) entries for individual plies [20]. Base surfaces are generally the tooling surfaces on which plies are laid, and the table specifies the order, materials and fiber directions of the plies. Obtaining a reduced model amounts to replacing the specified layup table from having \(n\) entries by a simpler table with just 3 entries.

To establish the validity of the proposed reduced laminate model, we tested it on several simple but common benchmark laminate problems. For simplicity, we assumed plate behavior everywhere in the laminate without any distinction between plate and non-plate regions so that we can replace the entire original laminate with a 3-ply laminate. This assumption is justified because we compare our results with those obtained using dimensionally reduced models of laminates.

The reduced model was fully implemented in a meshfree system called \textit{Scan and Solve} (SnS), which approximates displacements and stresses using multi-variate B-spline functions that are constructed over a uniform Cartesian grid (see [21] for details). This choice of basis addresses the concerns of shear locking and numerical ill-conditioning of stiffness matrix for a wide range of thicknesses. When a (surface representing a) ply intersects the support of a B-spline function, computation of the corresponding stiffness matrix coefficient requires integration over the ply. Assuming that the thickness of the ply is constant, this task reduces to integration over the ply’s surface. Lastly, we note that in a meshfree approach, the coordinate system of an element is generally not aligned with the \(x, y, z\) directions assumed in deriving the constitutive material model of the composite laminate. The discrepancy is easily corrected by a suitable coordinate transformation during the integration process.

4 Result and Verification

In this section, we compare results using proposed method to known results from literature and to results computed by commercial software SolidWorks [22]. Laminates considered are cross-ply laminates (0/90/0/90/...) and angle-ply laminates (-45/45/-45/45/...). For both the cases, when the plies are in even number, there is an asymmetry about the mid-plane leading to stretching-bending coupling.

Certain parameters are constant for all the tests. We use around 1000 second-order b-spline basis functions and 4 integration surfaces per ply for all the cases below. SolidWorks uses dimensionally reduced parabolic triangular shell elements in all the cases but the number of elements varies.

4.1 Flat Plate

First, we tested a flat plate that was 10\(\times\)10\(m^2\) wide and 0.1\(m\) thick, and under a normal load of \(10^4\)N on the right end with the left end fixed (Figure 4). Material properties of the plies in principal directions are: \(E_1 = 133.86\)GPa, \(E_2 = E_3 = 7.706\)GPa, \(v_{12} = .301\), \(v_{23} = v_{13} = 0\), \(G_{12} = G_{13} = 4.3\)GPa and \(G_{23} = 2.76\)GPa.

For different types of laminates, Table 1 compares the maximum total displacement from SolidWorks (SW) to values from our method (SnS). The results show close agreement between the two methods. While SnS used 1000 elements, SolidWorks is using 648 elements with total 1369 nodes or degrees of freedom.

To test if our method works for a large number of plies, we analyzed the same plate but made of 50 random plies (Appendix A). Maximum displacement from our method was \(5.781e^{-5}\)m and from SolidWorks was \(2.176e^{-5}\)m, which are substantially off. This disparity led us to test the same laminate in another commercial system ANSYS [23]. In ANSYS, we used four node shell elements called \textit{SHELL181} that is based on first order shear deformation theory. The maximum displacement from ANSYS was \(5.73e^{-5}\), which is close to the displacement value from our method. Figure 4 shows the color-map of total displacement from our method as well as ANSYS.

Using the proposed method, the total time spent in analyzing the 50 ply laminate plate was 22.5 seconds, out of which almost 19 seconds were spent doing integration over the 12 integration surfaces. Without the reduced model, integrating over 50 plies will take at least 75 seconds. So the proposed method has decreased the computation cost by almost 4 times, and the efficiency will increase with the number of plies.
4.2 Clamped Cylinder With Internal Pressure

Next, we compare results for a clamped cylindrical shell with internal pressure $P_o$. Forces and boundary conditions are shown in Figure 5. Material properties of the plies are: $E_1 = 7.5 \times 10^6 \text{psi}$, $E_2 = E_3 = 2 \times 10^6 \text{psi}$, $\nu_{12} = .25$, $\nu_{23} = \nu_{13} = 0$, $G_{12} = 1.25 \times 10^6 \text{psi}$, and $G_{13} = G_{23} = 0.625 \times 10^6 \text{psi}$.

Table 5 compares maximum radial deflection from [4], SolidWorks, and our method. Reddy [4] used 16 Q4 elements, a four-node (linear) quadrilateral element, and 4 Q9 elements, a nine-node (quadratic) quadrilateral element. In SolidWorks, we modeled the cylinder using 1206 elements with 1538 nodes in total, while Scan and Solve used 1000 elements. Again we see a consistent agreement between all the methods.

4.3 Barrel Vault Problem

Finally, we consider a popular benchmark shell problem known as the Barrel vault problem [4]: a cylindrical roof under its own weight. Detailed boundary conditions are shown in Figure 6. Material properties of the plies are: $E_1 = 25 \times 10^6 \text{psi}$, $E_2 = E_3 = 1 \times 10^6 \text{psi}$, $\nu_{12} = .25$, $\nu_{23} = \nu_{13} = 0$, $G_{12} = G_{13} = 5 \times 10^5 \text{psi}$ and $G_{23} = 2 \times 10^5 \text{psi}$.

Tables 3 and 4 compare maximum deflections for different cross-ply and angle-ply laminates. To test if our method works for shells of different thicknesses, we compared results for both thin and thick shells. Number of shell elements used by SolidWorks is 988 with 2067 nodes, while Reddy [4] uses 16 Q81 ele-
FIGURE 6. Figure showing barrel vault problem. Vertical pressure is \( q = 625 \text{psi} \) and the curved ends are fixed. Other dimension: \( \beta = 80^\circ \), \( R = 300 \text{in} \), \( a = 600 \text{in} \), \( h = 3, 6 \) and \( 15 \text{in} \)

TABLE 3. Barrel vault problem- maximum deflection(in.) for cross-ply laminates

<table>
<thead>
<tr>
<th>Laminate</th>
<th>( S=R/h )</th>
<th>Reddy</th>
<th>SW</th>
<th>SnS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 plies</td>
<td>100</td>
<td>2.34</td>
<td>2.46</td>
<td>2.27</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.508</td>
<td>0.566</td>
<td>0.537</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.0729</td>
<td>0.0756</td>
<td>0.0773</td>
</tr>
<tr>
<td>10 plies</td>
<td>100</td>
<td>1.42</td>
<td>1.56</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.294</td>
<td>0.327</td>
<td>0.327</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.0523</td>
<td>0.0535</td>
<td>0.0529</td>
</tr>
</tbody>
</table>

TABLE 4. Barrel vault problem- maximum deflection(in.) for angle-ply laminates

<table>
<thead>
<tr>
<th>Laminate</th>
<th>( S=R/h )</th>
<th>Reddy</th>
<th>SW</th>
<th>SnS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 plies</td>
<td>100</td>
<td>3.6</td>
<td>3.87</td>
<td>3.32</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.676</td>
<td>0.717</td>
<td>0.635</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.121</td>
<td>0.113</td>
<td>0.105</td>
</tr>
<tr>
<td>10 plies</td>
<td>100</td>
<td>1.82</td>
<td>1.96</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.41</td>
<td>0.409</td>
<td>0.373</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.1</td>
<td>0.09</td>
<td>0.0779</td>
</tr>
</tbody>
</table>

ments, elements with \( p \) level 8, and 405 degrees of freedom. For our method, elements are same as before. Again, there is a agreement between results from [4], SolidWorks, and our method.

5 Conclusion and Future Work

We proposed a new reduced material model for laminates that significantly decreases the computational cost of performing 3D FEA on composite laminates. The reduction was achieved using standard lamination and plate theories. The new model is formally equivalent to the popular ABD model used in 2D analysis of laminates and is valid anywhere in the structure where the same underlying assumptions apply. This reduced model can be implemented in any 3D FEA system which supports laminates, effectively taking advantage of dimensional reduction while still using general 3D finite elements. We also demonstrated that with a suitable choice of basis functions, the reduced model makes 3D FEA practical and efficient enough to be considered an attractive alternative to existing semi-automated methods.

We validated the proposed reduced material model by showing that its use in 3D meshfree analysis system leads to results that are consistently in good agreement with known results for several benchmark problems. Our future work will focus on incorporating the reduced material model in analysis of complex composite structures which consist of multiple laminates, joints, and discontinuities. The immediate challenges include identifying regions in the laminate where the reduced model can be used and efficient handling of the regions where this reduced model cannot be used.

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REFERENCES


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Appendix

A 50 Random plies
The following fiber orientations were assigned to 50 random plies used in the flat plate testing: -45, 45, 0, -45, 90, 45, -50, -75, 60, -45, 90, -45, -45, 45, -45, -75, -5, 80, 30, -45, -45, 60, 90, -75, -45, 25, -45, -45, 45, -45, -75, 60, 60, -45, 90, -45, -45, -75, -50, 45, -45, 60, -45, 50, -75, -45, -75, 10, -45, 60, 90